CONSTRUCTION OF (L-R) TYPE NUMBERS BASED ON FUZZY LINGUISTIC ASSESSMENTS

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Introduction

Recently, fuzzy modelling has become one of the most active and promising directions in applied research. In fuzzy modelling, fuzzy numbers are most often used to represent fuzzy sets. They serve as the basis for constructing mathematical models and allow setting fuzzy values and performing arithmetic operations with them.

In practice, operations over fuzzy numbers and (L-R) type intervals have gained widespread use, the application of which reduces the computational workload. Moreover, (L-R) numbers can be used to define intervals of parameter values whose exact boundaries are challenging to set under conditions of uncertainty.

The issues of (L-R) approximation have been examined by many researchers. However, despite the extensive range of studies on various aspects of (L-R) approximation, there remains a need for the development of powerful yet simple ways to approximate (L-R) numbers themselves. This report proposes one of the possible approaches, based on fuzzy linguistic assessments.

Fuzzy linguistic assessments

A fuzzy linguistic assessment is understood as a numerical evaluation expressed in statements with the quantifiers "approximately/about":

"The parameter value is approximately c" or

"The parameter value is approximately in the range from c to d".

Fuzzy numbers of the (L-R) type can be approximated by fuzzy linguistic assessments. Triangular numbers (c, α, β) by the assessments of the first type, and trapezoidal numbers (c, d, α, β) by the assessments of the second type. Since the values c and d in these numbers are specified by linguistic assessments, the task of constructing such numbers consists only in determining the fuzziness coefficients α and β.

Calculation of Fuzziness Coefficients

The fuzziness coefficients determine the boundaries of the support of the fuzzy set $\tilde{A}$ i.e., the points where the membership function is usually $\mu_{\tilde{A}}(x) = 0.01$..

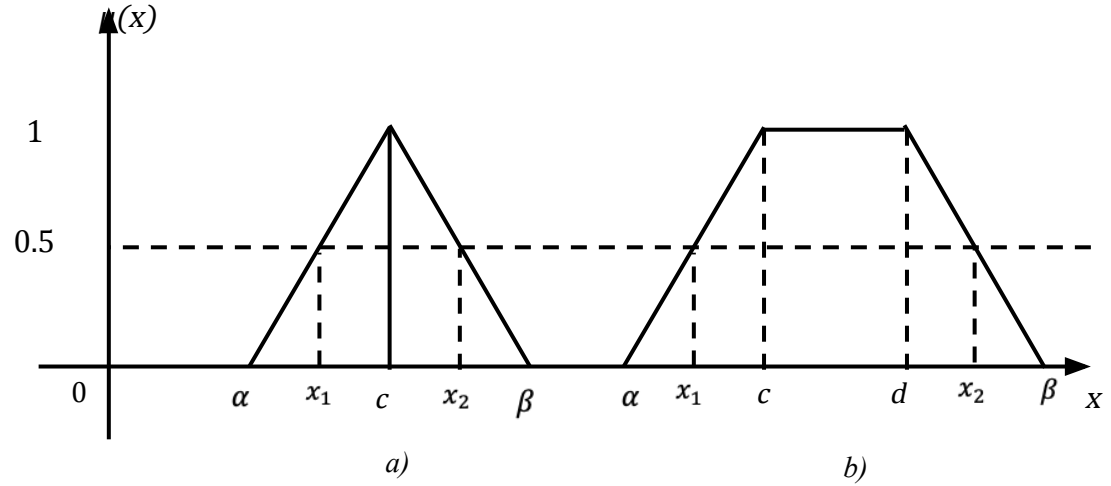

Figure 1: Membership functions: a) - triangular; b) – trapezoidal

Knowing the equations of the left function $f_L(x)$ and the right $f_R(x)$, the fuzziness coefficients are calculated as follows: $\alpha = f_L^{-1}(0.01)$, $\beta = f_R^{-1}(0.01)$.. For the triangular membership function, the equation$f_L(x)$ is determined by the points $(x_1, 0.5)$, $(c, 1)$ and the equation $f_R(x)$ by the points $(c, 1), (x_2, 0.5)$, where x_1 and x_2 are the transition points..

Given b(c) is the distance between the transition points of the triangular membership function. Then: $x_1 = c - \frac{d}{2}$ and $x_2 = c + \frac{d}{2}$.

For the trapezoidal membership function, the distances $b(c)$ and $b(d)$ are first calculated, and the points x_1 and x_2 are determined as: $x_1 = c - \frac{b(c)}{2}, x_2 = d + \frac{b(d)}{2}$. Then, like the triangular function, equations of the lines$f_L(x)$ and $f_R(x)$ are constructed, passing through the $(x_1, 0.5), (c, 1)$ and $(d, 1), (x_2, 0.5)$ respectively. From these equations, the fuzziness coefficients α, β are calculated.

Determination of the distance between transition points

1, 2, 3, 4, 6, 7, 8, 9	
10, 20, 30, 40, 60, 70, 80, 90	
35, 45, 55, 65, 75, 85, 95 5 15 25 50 Other two-digit numbers	

If $T \in [1,99]$ then $b(T)$ is determined according to this table. Otherwise, the following algorithm is used.

Algorithm for calculating the distance between transition points

Let a fuzzy set be defined as "the number is around T ", where T is a natural number. If $T \in [1,99]$, then $b(T)$ is determined according to Table 1. Otherwise, let its least significant digit have the order of q. Possible values of q are divided into residue classes modulo 3. As a result, three classes $M_d, d \in \{0,1,2\}$ are obtained, where $d = q\ mod(3)$. In this case, the value $b(T)$ also depends on the class M_d to which the number T belongs.

Let r_q be the digit in the q place of the number T. Then:

1. if $T \in M_0$, then $b(T) = b(x) \cdot 10^{q-2}$, where $x = r_q \cdot 10$, and $b(x)$ is taken from Table 1.
2. if $T \in M_1$, then there are two options:
   a) if $r_{q+1} = 0$, then $b(T) = b(x) \cdot 10^{q-1}$, where $x = r_q$;
   b) if $r_{q+1} \neq 0$, then $b(T) = b(x) \cdot 10^{q-1}$, where $x = r_{q+1} \cdot 10 + r_q$.
3. if $T \in M_2$, then there are also two options:
   a) if $r_{q+1} = 0$, then $x = r_q \cdot 10$; $b(T) = b(x) \cdot 10^{q-2}$;;
   b) if $r_{q+1} \neq 0$, then $x = r_{q+1} \cdot 10 + r_q$; $b(T) = b(x) \cdot 10^{q-1}$.

As a result, the value $b(T)$ of will be obtained..

Building fuzzy numbers using Gaussian functions

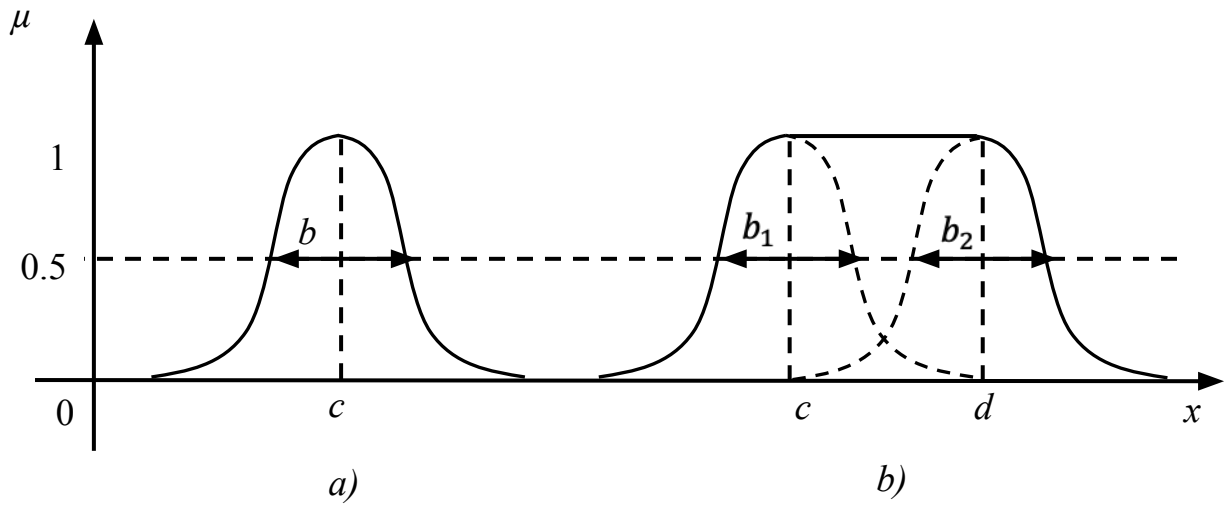

Figure 2: Gaussian membership functions: a) - standard; b) - combined (double)

The standard Gaussian function is used to define fuzzy sets $\tilde{A} \triangleq$ corresponding to "the number is approximately equal to c". In this case, the function is given by: $\mu_{\tilde{A}}(x) = exp(-a(x-c)^2)$, where $a = -\frac{4\ ln\ 0.5}{b^2(c)}$ and $b(c)$ distance between transition points..

In this case, the coefficients α and β are derived from the equation $\mu_{\tilde{A}}(x) = 0.01$. These coefficients can also be approximately calculated as $\alpha = c - \frac{k \cdot b(c)}{2}$, and $\beta = c + \frac{k \cdot b(c)}{2}$, where $k \approx 2.5$ is a scaling coefficient.

The combined Gaussian function

The combined function describes the fuzzy set $\tilde{A} \triangleq$ "the number is approximately in the interval from ***c*** to ***d***". This function takes the form:

$$\mu_{\tilde{A}}(x) = \begin{cases} x < c, & \mu_{\tilde{B}}(x) \\ c \le x \le d, & 1 \\ x > d, & \mu_{\tilde{C}}(x) \end{cases},$$

where $\mu_{\tilde{B}}(x)$ is the membership function of the fuzzy set $\tilde{B} \triangleq$ "the number is around ***c***", and $\mu_{\tilde{C}}(x)$ is the membership function of the fuzzy set $\tilde{C} \triangleq$ "the number is around ***d***". In this case, the fuzziness coefficients α and β can be found from equations $\mu_{\tilde{B}}(x) = 0.01$ and $\mu_{\tilde{C}}(x) = 0.01$. As a result, a fuzzy interval $M_2 = (c, d, \alpha, \beta)$ is obtained.

Conclusion

An approach to constructing fuzzy numbers and intervals based on fuzzy linguistic statements has been proposed. Depending on the type of statements, triangular or trapezoidal fuzzy numbers of (L-R) type are automatically constructed. This allows the use of this approach when specifying model parameters and simulating random variables, representing fuzzy time series in a verbal form, forming databases in fuzzy inference systems, information queries, as well as in many other applied aspects under conditions of uncertainty.