# Upper bound of buffer content distribution for self-similar traffic models

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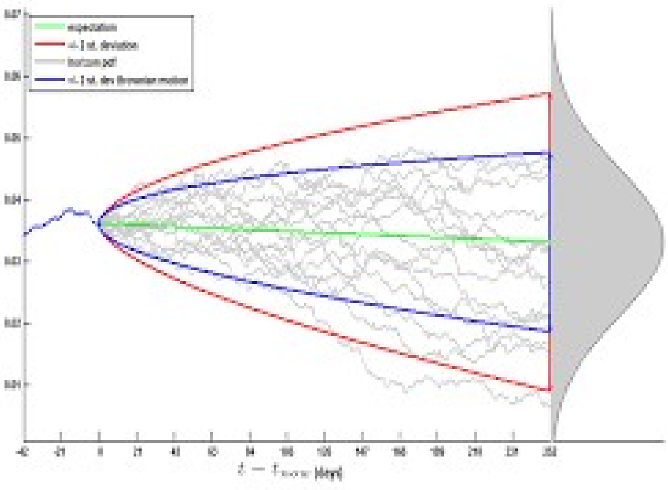

# 1. Introduction

- The information content has increased seriously with regard to the steady development of communications and telecommunications and the emergence of new types of services.

- Classical distributions are not always suitable to describe the existing flows in modern networks.

- New approaches and types of distributions are used to analyze traffic characteristics, and their study sometimes cannot be studied analytically.

- Researches show that the traffic has fractal structure. It has the properties of self-similarity and long-term dependence.

- One of the main parameters of multifractal traffic is the Hurst index which can help to determine the degree of long-term dependence (the rate of decrease of the correlation function).

We consider self-similar traffic models that can be described by FBM. The arrival and backlog processes connected with FBM are defined and the main properties are studied.

The estimation of probability buffer overflow is obtained. We also analyze a traffic that enters as input on some time-invariant linear system.

To provide the connection of input traffic and the response (output) of such system a quadratic form built on these two processes is considered. We find the upper bound of buffer content distribution taking into account input and output traffic.

A particular case of self-similar traffic model is observed and the dependence of model parameters is studied.

# 2. Stochastic models in telecommunication

In the field of telecommunications it is often used stochastic models. Discrete and fluid queueing models are played a great role in the development of computer and communication networks.

it is very important to consider certain performance issues known as **Quality-of-Service (QoS).** There are some well-known end-to-end QoS measures, for example,

- *loss probability,*
- *delay,*
- *delay-jitter and*
- *bandwidth*.

Telecommunication networks are typically hierarchical in nature. Although, different researchers prefer to use different traffic models, the models can be broadly classified into two parts, discrete models and fluid models.

## 2.1. Self-similarity in traffic models

One of the most important findings of traffic measurement studies over the last years is the observed self-similarity in packet network traffic. A lot of researches has focused on the origins of this self-similarity, and the network engineering significance of this phenomenon, see for example.

A good starting point in understanding the impact of self-similarity was provided by Norros, who developed a formula that can be used to estimate buffer overflow probabilities at network switches and routers. The Norros results showed that the queueing backlogs were in general worse with self-similar traffic, in the sense that the buffer sizes to achieve a certain loss objective could be significantly greater.

There are three fundamental conditions that should be satisfied for a Gaussian self-similar traffic, corresponding to the observation of long-range dependence in traffic, to apply:

1) the network traffic should be sufficiently aggregated so that the marginal distributions of counts are at least approximately Gaussian (due to Central Limit Theorem);
2) the long-range dependent scaling region should span the engineering time scales of interest;
3) the impact of network controls on the traffic flows must not be significant over the engineering time scales of interest.

These three conditions together suggest a feasibility regime for the standard self-similar traffic model based on fractional Brownian motion (FBM).

## 3. Arrival and backlog processes

To describe the character of the observed properties of traffic data more precisely, we introduce a second order stochastic process.

Let $(\Omega, \mathrm{B}, P)$ is a standard probability space.

**Definition 1.** We say that stochastic process $B_\alpha(t), t \in [0,1]$, is called the generalized Wiener process (fractional Brownian motion, FBM) with the Hurst index $\alpha \in (0,1)$ if the following conditions hold true:

1. it's Gaussian stochastic process;
2. starts at zero, $B_\alpha(0) = 0$,
3. with zero expectation $EB_\alpha(t) = 0$ and
4. it has a covariance function $R_\alpha(t,s) = \frac{1}{2}\left(|t|^{2\alpha} + |s|^{2\alpha} - |t-s|^{2\alpha}\right)$.

The self-similarity parameter $\alpha \in (0,1)$, Hurst index, has the following role.

If $\alpha \neq \frac{1}{2}$ then the process $B_\alpha(t)$, is a process with dependent increments. There are $\frac{1}{2}$-self-similar processes with independent increments.

In the case $\alpha < \frac{1}{2}$ the increments of FBM are negatively correlated. In contrast, when $\alpha > \frac{1}{2}$ the increments of FBM is positively correlated and the increments of the process $B_\alpha(t)$ are long-range dependent. The case $\alpha < \frac{1}{2}$ corresponds to short-range dependence.

Under $A(t)$ let us denote the the arrival traffic, e.i. amount of traffic coming to the network over a period of time $[0,T]$. The increment we denote as $A(s,t) = A(t) - A(s), \quad t > s > 0$. In [5] it's shown that input traffic ca be presented as

$$A(t) = mt + \sqrt{am} B_\alpha(t),$$

where $m$ is an average traffic rate, $B_\alpha(t)$ is FBM with Hurst index $\alpha$, $a$ is some constant.

If the network has one service device with the rate $C > m$, the backlog process can be defined as

$$Q(t) \cong \sup_{s \le t} \left( A(s,t) - C(t-s) \right).$$

The system with $n$ independent identically service devices provides the backlog processes as

$$Q_n(t) \cong \sup_{s \le t} \left( \sum_{i=1}^{n} A_i(s,t) - nC(t-s) \right).$$

Here, the symbol $\cong$ means identically distributed quantity.

Study now the probability of overloading by threshold $b$ of $Q(t)$ on time interval $[0,T]$. Let

$$Q \cong \sup_{t \in [0,T]} \left( Q(t) \right), \quad \pi(b) = P\{Q \ge b\}.$$

We are interested in the upper bound for buffer content distribution

$$P\{Q \ge b\} \le P\left\{ \sup_{t \in [0,T]} \left( Q(t) \right) > b \right\}.$$

Really, it was shown that $P\{Q \ge b\} \le P\left\{ \sup_{t \in [0,T]} \left( |B_\alpha(t)| \right) > \dfrac{b + T(C-m)}{2\sqrt{am}} \right\}$.

Let us denote $x = \frac{b + T(C - m)}{2\sqrt{am}}$, then the following theorem is fulfilled.

**Theorem 1.[17]** For $x \geq D$ we have

$$P\left\{ \sup_{t \in [0,T]} \left( \left| B_\alpha(t) \right| \right) > x \right\} \leq 2\exp\left\{ -\frac{(x - D)^2}{2V} \right\} \tag{1}$$

**Remark 1.** Using the estimates obtained, we can determine what the threshold should be, so that the probability of being exceeded is less than given.

Thus,

$$P\{Q \geq b\} \leq P\left\{ \sup_{t \in [0,T]} \left( Q(t) \right) > b \right\} \leq P\left\{ \sup_{t \in [0,T]} \left( \left| B_\alpha(t) \right| \right) > x \right\} \leq \varepsilon_b, \text{ if}$$

$$x \geq D \text{ and } 2\exp\left\{ -\frac{(x - D)^2}{2V} \right\} \leq \varepsilon_b.$$

And the threshold should be $b \geq 2D\sqrt{am} - T(C - m)$.

The value $\varepsilon_b$ one can interpretate as significance level.

# 4. Square-Gaussian stochastic processes

Assume that $(T, \rho)$ is some compact metric space with metric $\rho$.

**Definition 2.** Let $\Xi = \{\xi_t, t \in T\}$ be a family of zero-mean joint Gaussian random variables. A space $SG_\Xi(\Omega)$ is a space of Square-Gaussian random variables if any element of this space $\eta \in SG_\Xi(\Omega)$ can written as

$$\eta = \zeta A \zeta^T - E\zeta A \zeta^T,$$

where $\zeta = (\xi_1, \xi_2, ..., \xi_n)$, $\xi_k \in \Xi$, $k = \overline{1, n}$, $A$ is a real-valued matrix or an element, $\eta \in SG_\Xi(\Omega)$ is a square mean limit of the sequence $\eta = \underset{n\to\infty}{\text{l.i.m}}(\zeta_n A \zeta_n{}^T - E\zeta_n A \zeta_n{}^T)$.

**Definition 3.** A stochastic process $\xi(\mathrm{t}), \mathrm{t} \in [0, \mathrm{T}]$, is called Square-Gaussian if for any fixed t ϵ [0,T] each random variable $\xi(\mathrm{t})$ belongs to the space $SG_\Xi(\Omega)$ and $\sup\limits_{t\in[0,T]} |\xi(t)| < \infty$.

**Theorem 2**. [21] Assume that $\xi(t), t \in [0,T]$, is a separable Square-Gaussian stochastic process and

$$\sup_{|t-s|<h} \sqrt{D(\xi(t)-\xi(s))} \le \sigma(h) = kh^{\beta}, \ \alpha \in (0,1], \quad (2)$$

where $k$ is some constant. Then for x such that

$$x > \frac{2\sqrt{2}\max\{\delta_0, k(\mathrm{T}/2)^{\beta}\}}{\beta},$$

the inequality

$$P\left\{\sup_{t\in[0,T]} |\xi(t)| > x\right\} < 4e^{\frac{3}{\beta}} \exp\left\{-\frac{x}{2\sqrt{2}\delta_0}\right\} \times \left(\frac{x\beta}{2\sqrt{2}\delta_0}\right)^{2/\beta} \left(1+\frac{2x}{\sqrt{2}\delta_0}\right)^{1/2}$$

holds true where $\delta_0 = \sup_{t\in[0,T]} (D(\xi(t)))^{1/2}$.

# 5. Upper bound of buffer content distribution taking into account input and output traffics of stochastic system

In the case of traffic signal transfer on some system not only input process but also the response (output) of system should be taken into account

Consider a time-invariant linear system with a real-valued square integrable impulse response function $H(\tau)$ which is defined on a finite domain $\tau \in [0, \mathrm{T}]$. This means that the response of the system to an input signal $X(\mathrm{t})$ which is observed on $[-\mathrm{T}, \mathrm{T}]$ has the following form

$$Y(\mathrm{t}) = \int_0^T H(\tau)\, \mathrm{X}(\mathrm{t} - \tau)\, \mathrm{d}\,\tau, \quad t \in [0, \mathrm{T}] \tag{3}$$

and $H \in L_2([0, \mathrm{T}])$.

In [14] we can find that Fractional Brownian Motion can be expanded in the form of random series

$$B_\alpha(t) = X_\alpha(t) = \sum_{k=1}^{\infty} \left( a_k \sin\left(x_k t\right) X_k + b_k \left(1 - \cos\left(y_k t\right)\right) Y_k \right)$$

where $\{X_k, Y_k\}$ are uncorrelated standard Gaussian random variables, $\{x_k\}$ are zeros of Bessel function $J_{-\alpha}(x)$ and $\{y_k\}$ zeros of Bessel function $J_{1-\alpha}(x)$,

$$a_k = \frac{\pi^{\alpha}\sqrt{2C}}{x_k^{\alpha+1} J_{1-\alpha}(x_k)}, \quad b_k = \frac{\pi^{\alpha}\sqrt{2C}}{y_k^{\alpha+1} J_{-\alpha}(y_k)}, \quad C = \frac{\Gamma(2\alpha+1)\sin(\pi\alpha)}{\pi^{2\alpha+1}}.$$

Suppose that the impulse response function is known. We also suggest that the input signal in system (3) is FBM with Hurst index $\alpha$. From (3) follows that the response of the system (output) $Y(\mathrm{t})$ can be presented as

$$Y(t) = Y_{\alpha}(\mathrm{t}) = \sum_{k=1}^{\infty} (\xi_k \cdot \mathrm{c}_k(t) + \eta_k \cdot \mathrm{s}_k(t)), \tag{4}$$

where the functions $c_k(\mathrm{t})$, $\mathrm{s}_k(\mathrm{t})$ equal

$$\mathrm{c}_k(\mathrm{t}) = b_k \int_0^T H(\tau)(1-\cos(\mathrm{y}_k(\mathrm{t}-\tau)))\,\mathrm{d}\tau,$$

$$s_k(\mathrm{t}) = a_k \int_0^T H(\tau)\sin(\mathrm{x}_k(\mathrm{t}-\tau))\,\mathrm{d}\tau. \tag{5}$$

In this section we investigate the backlog of system with input signal FBM $X_{\alpha}(\mathrm{t})$, taking into account the output of the system $Y(\mathrm{t})$.

Under $\xi\,(t)$ we denote the sum of square $X_\alpha(\mathrm{t})$ and $Y_\alpha(\mathrm{t})$

$$\xi\,(t) = (X_\alpha(\mathrm{t}))^2 + (Y_\alpha(\mathrm{t}))^2 \,.$$

Making the same manipulation as in [17] the probability of backlog can be estimated as

$$P\{Q \ge b\} \le P\left\{ \sup_{t\in[0,T]} \left( \sqrt{X^2{}_\alpha(t) + Y_\alpha^2(t)} \right) > \frac{b+T(C-m)}{2\sqrt{am}} \right\} =$$

$$= P\left\{ \sup_{t\in[0,T]} \left( X^2{}_\alpha(t) + Y_\alpha^2(t) \right) > \left( \frac{b+T(C-m)}{2\sqrt{am}} \right)^2 \right\} =$$

$$= P\left\{ \sup_{t\in[0,T]} |\, \xi(t) - E\xi(t)\, | > x_0 \right\},$$

where $x_0 = \left( \frac{b+T(C-m)}{2\sqrt{am}} \right)^2 + \sup_{t\in[0,T]} E\xi(t)$.

Let's make the following notation:

$$\phi^1_{kl} = \phi^1_{kl}(\mathrm{t}) = \mathrm{b}_k\, a_l(1-\cos(\mathrm{y}_k\, t))(1-\cos(\mathrm{y}_l\, t)) + c_k(\mathrm{t})\,\mathrm{c}_l(\mathrm{t});$$

$$\phi^2_{kl} = \phi^2_{kl}(\mathrm{t}) = 2(\mathrm{b}_k\, a_l(1-\cos(\mathrm{y}_k\, t))\sin(\mathrm{x}_l\, t) + c_k(\mathrm{t})\,\mathrm{s}_l(\mathrm{t}));$$

$$\phi^3_{kl} = \phi^3_{kl}(\mathrm{t}) = a_k a_l \sin(\mathrm{x}_k\, t)\sin(\mathrm{x}_l\, t) + s_k(\mathrm{t})\,\mathrm{s}_l(\mathrm{t}).$$

Then by (3), (4) we have that quadratic form $\xi(t)$ can be written as

$$\xi\ (\mathrm{t}) = \sum_{k=1}^{\infty}\sum_{l=1}^{\infty}(\phi^1_{kl}(\mathrm{t})\xi_k\xi_l + \phi^2_{kl}(\mathrm{t})\xi_k\eta_l + \phi^3_{kl}(\mathrm{t})\eta_k\eta_l).$$

Let us also denote the increments of the functions

$$\Delta\phi^1_{kl} = \phi^1_{kl}(\mathrm{t}) - \phi^1_{kl}(\mathrm{s});\quad \Delta\phi^2_{kl} = \phi^2_{kl}(\mathrm{t}) - \phi^2_{kl}(\mathrm{s});$$

$$\Delta\phi^3_{kl} = \phi^3_{kl}(\mathrm{t}) - \phi^3_{kl}(\mathrm{s}).$$

At first, present the auxiliary relationships concerning mean, variance and variance of increments for the process $\xi\ (t)$.

**Lemma 1.** The mean, variance and variance for the increments of stochastic form $\xi(t)$ equal:

$$E\xi(t)=\sum_{k=1}^{\infty}(\phi_{kk}^{1}(t)+\phi_{kk}^{3}(t));$$

$$D\xi(t)=\sum_{k,l=1}^{\infty}\left(2(\phi_{kl}^{1}(t))^{2}+(\phi_{kl}^{2}(t))^{2}+2(\phi_{kl}^{3}(t))^{2}\right);$$

$$D(\xi(t)-\xi(s))=\sum_{k,l=1}^{\infty}\left(2(\Delta\phi_{kl}^{1})^{2}+(\Delta\phi_{kl}^{2})^{2}+2(\Delta\phi_{kl}^{3})^{2}\right).$$

If we put $d_{kl}=\sup\limits_{t\in[0,T]}(2(\phi_{kl}^{1}(t))^{2}+(\phi_{kl}^{2}(t))^{2}+2(\phi_{kl}^{3}(t))^{2})$. Then $\sqrt{D\xi(t)}\le\left(\sum\limits_{k,l=1}^{\infty}d_{kl}\right)^{1/2}:=\delta_0$.

Under some conditions it could be shown that

$$(D(\xi(t)-\xi(s)))^{1/2}\le K\cdot|t-s|^{\beta},\quad \beta\in(0,1],$$

where $K$ is a some constant.

**Theorem 3**. Suppose that the input traffic of system (3) is FBM $X_\alpha(t)$. If

$$x_0 > \frac{2\sqrt{2}\max\{\delta_0, K(\mathrm{T}/2)^\beta\}}{\beta},$$

then the inequality

$$P\{Q > b\} < 4e^{\frac{3}{\beta}} \exp\left\{-\frac{x_0}{2\sqrt{2}\delta_0}\right\}\left(\frac{x\beta}{2\sqrt{2}\delta_0}\right)^{2/\beta}\left(1+\frac{2x_0}{\sqrt{2}\delta_0}\right)^{1/2}.$$

Using the obtained results, it is possible to estimate the required buffer size in the framework of different measurement options. The estimation is performed with given accuracy and reliability.

# 6. Case study

In this section we will investigate the dependence of the Hurst index and the threshold value in backlog process, the dependence of the service characteristics such as rate and the value of the threshold.

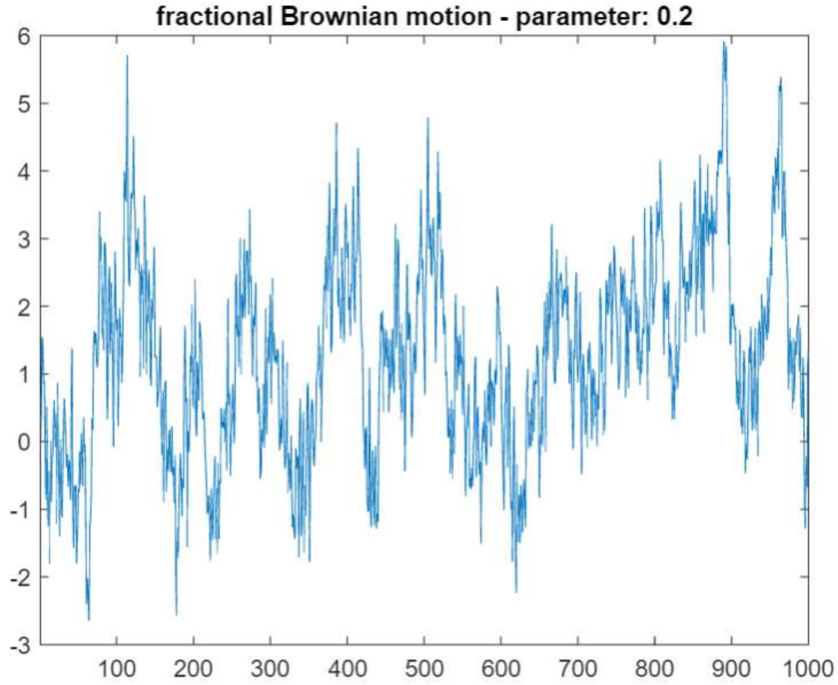


**Figure 1**: The sample of FBM with Hurst index 0.2

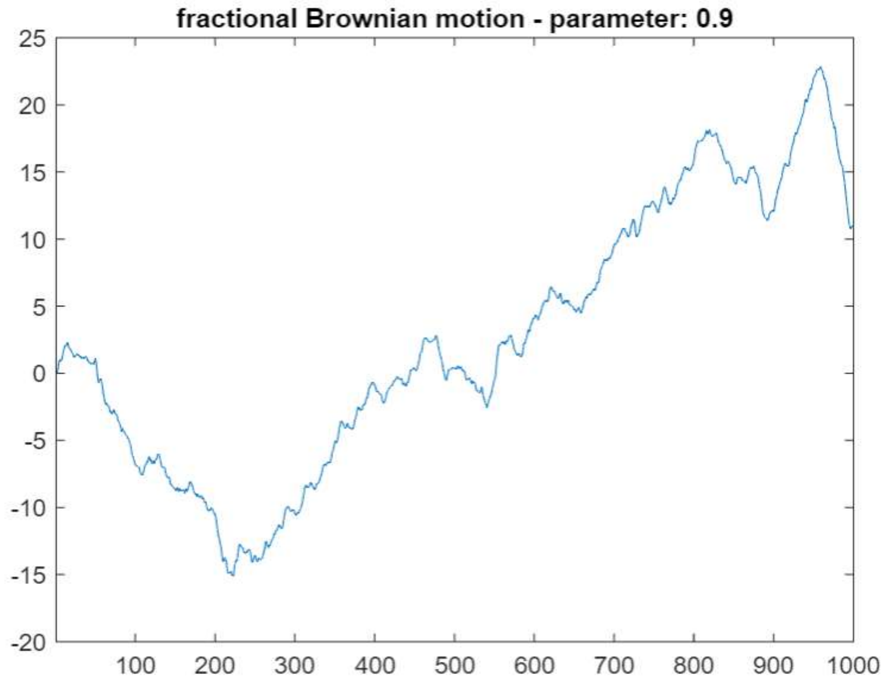


**Figure 2**: The sample of FBM with Hurst index 0.9

Let's consider the dependencies of the upper bound of buffer content and simulation on different traffic characteristics. Let us remind the notations that were used in this paper:

*m* is the average traffic rate,

*C* is the service rate (it is necessary that C>m),

*α* is the Hurst index,

*T* is the time period over which the service process is considered, and is a certain traffic coefficient,

the probability of traffic buffer overflow is less then a given $\varepsilon_b$ that can be considered as significance level.

Applying Theorem 1 and substituting the values of above described parameters: $\varepsilon_b = 0.05$; C = 100; m = 90; a = 1; T = 1; α = 0.9, we obtain the upper bound of traffic backlog threshold should be b >= 62,6. For parameters $\varepsilon_b = 0.05$; C = 100; m = 90; a = 1; T = 1; α = 0.2 it follows that b >= 131.2.

**Table 1**
**The dependence of threshold level on the Hurst index under unchanged other parameters**

| № | Significance level, $\varepsilon_b$ | Service rate, C | Traffic rate, m | Time period, T | Hurst index, H | Threshold, b |
|---|---|---|---|---|---|---|
| 1 | 0.05 | 100 | 90 | 1 | 0.1 | 189.4 |
| 2 | 0.05 | 100 | 90 | 1 | 0.2 | 131.2 |
| 3 | 0.05 | 100 | 90 | 1 | 0.3 | 106.0 |
| 4 | 0.05 | 100 | 90 | 1 | 0.4 | 91.4 |
| 5 | 0.05 | 100 | 90 | 1 | 0.5 | 81.8 |
| 6 | 0.05 | 100 | 90 | 1 | 0.6 | 74.9 |
| 7 | 0.05 | 100 | 90 | 1 | 0.7 | 69.8 |
| 8 | 0.05 | 100 | 90 | 1 | 0.8 | 65.8 |
| 9 | 0.05 | 100 | 90 | 1 | 0.9 | 62.6 |

Thus, changing the value of the Hurst index as an input argument (and not changing the value of the service rate *C* and the average rate of arrival traffic *m*), we have the following dependence, shown in Fig. 3. It's seen that the larger Hurst index is the smaller threshold value is.

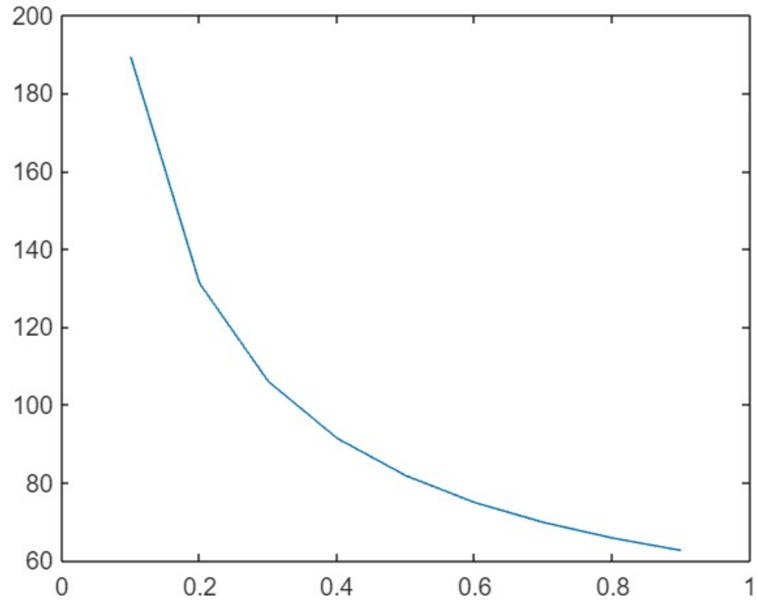


**Figure 3**: The functional relation of threshold and Hurst index

**Table 2**

**The dependence of threshold level on average traffic rate under unchanged other parameters**

| № | Significance level, $\varepsilon_b$ | Service rate, C | Traffic rate, m | Time period, T | Hurst index, H | Threshold, b |
|---|---|---|---|---|---|---|
| 1 | 0.05 | 100 | 99 | 1 | 0.9 | 75.1 |
| 2 | 0.05 | 100 | 90 | 1 | 0.9 | 62.6 |
| 3 | 0.05 | 100 | 80 | 1 | 0.9 | 48.4 |
| 4 | 0.05 | 100 | 70 | 1 | 0.9 | 34.0 |
| 5 | 0.05 | 100 | 60 | 1 | 0.9 | 19.3 |
| 6 | 0.05 | 100 | 50 | 1 | 0.9 | 4.1 |

The inequality in Theorem 1 can be analyzed in different ways. Table 2 shows how the changes in average traffic rate influences on the level of threshold. It is clear that such a dependence should be reversed, which is confirmed by obtained results.

# 7. Conclusions

We investigated self-similar traffic model driven by FBM. We defined the arrival and backlog processes of stochastic network and obtained estimation of upper bound probability buffer overflow. We also explored a traffic that enters as input on some time-invariant linear system and found the estimation of buffer content distribution taking into account input traffic and a response of stochastic system. Finally, we examined the obtained results in one particular case and showed how the threshold level of buffer overflow depends on such parameters of stochastic network as Hurst index and arrival traffic rate.

We see further research of the problem in the study of generalized FBM to consider traffic data.

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