MODELING OF RANDOM VARIABLES ON FUZZY INTERVALS OF THEIR VALUES

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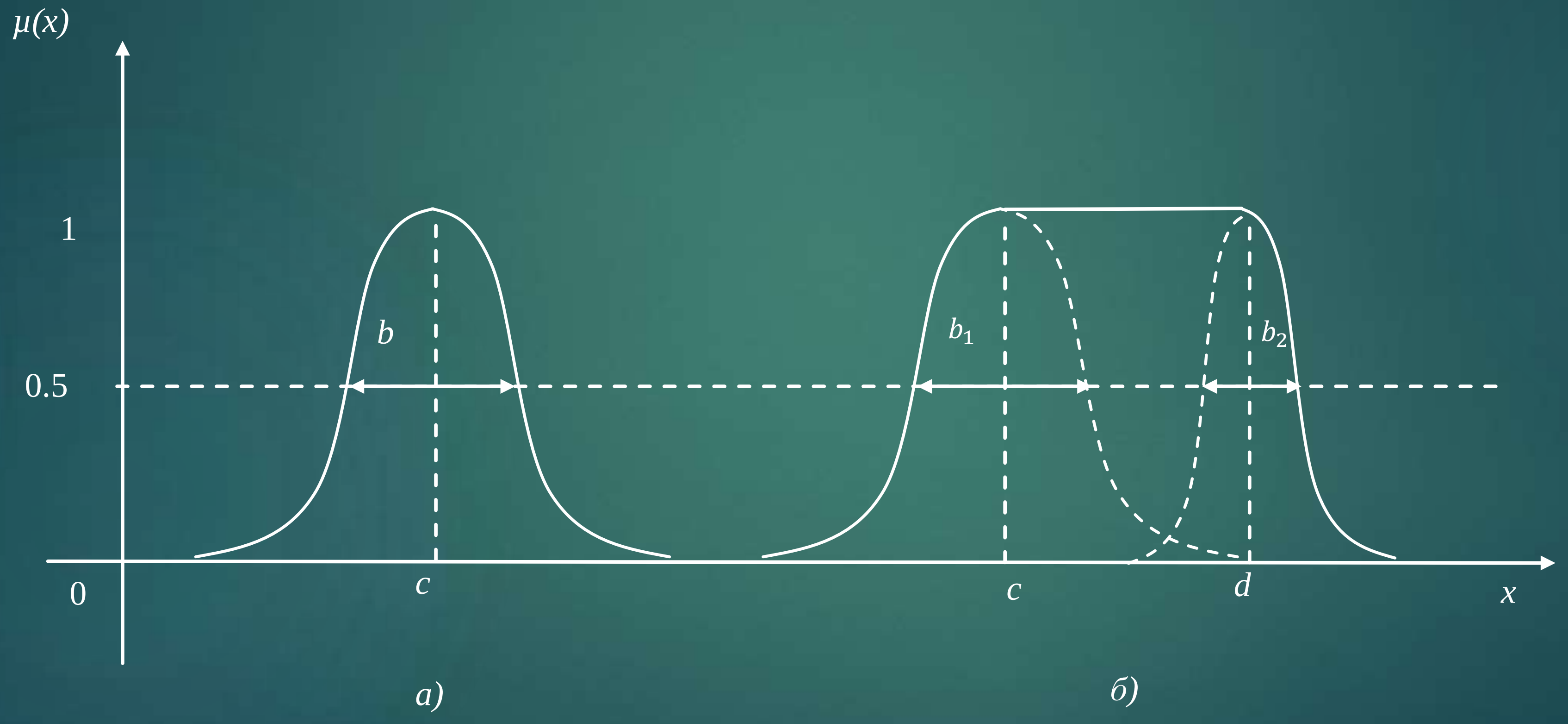
Representing fuzzy linguistic assessments fuzzy values



Gaussian membership functions a) - standard b) - combined (double).

Representing fuzzy linguistic assessments fuzzy values

The standard Gaussian function is used to define fuzzy sets $\tilde{C} \triangleq$ "the number is near c".

$$\mu_{\tilde{C}}(x) = \exp(-a(x-c)^2)$$

We will use the Gaussian function of the form:

$$\alpha = -\frac{4ln0.5}{b^2(c)}$$

where

$$\alpha = -\frac{4ln0.5}{b^2(c)}$$

The combined function describes fuzzy set $\tilde{A} \triangleq$"the number is approximately in the range from c to d".

Representing fuzzy linguistic assessments fuzzy values

- This function has the form:
- $\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{C}}(x), & x < c \\ 1, & c \leq x \leq d \\ \mu_{\widetilde{D}}(x), & x > d \end{cases}$

where $\mu_{\tilde{C}}(x)$ is the membership function of the fuzzy set $\tilde{C} \triangleq$"the number is near c", and $\mu_{\widetilde{D}}(x)$ is the membership function of the fuzzy set $\widetilde{D} \triangleq$ "the number is near d". These functions are built in a similar way.

Thus, depending on the type of fuzzy linguistic assessment of the interval of possible values of a random variable, its boundaries will be fuzzy coefficients α and β, respectively, of a fuzzy triangular or trapezoidal number.

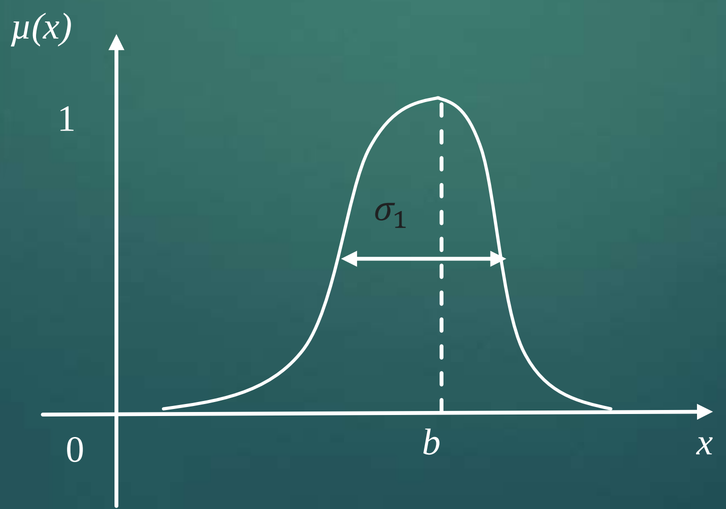
Calculating the Distance Between Transition Points

The distance between the transition points

Number x	**The distance between the transition points** $b(x)$
1, 2, 3, 4, 6, 7, 8, 9	$0{,}46\,x$
10, 20, 30, 40, 60, 70, 80, 90	$(0{,}357 - 0{,}00163x)x$
35, 45, 55, 65, 75, 85, 95	$(0{,}213 - 0{,}00067x)x$
5	2,8
15	6,48
25	6,75
50	24
Other two-digit numbers	$\frac{1}{2}(b\left(\left[\frac{x}{10}\right]\cdot 10+5\right)+b(x-\left[\frac{x}{10}\right]\cdot 10))$

Modeling of random variables

- *Case 1*. Let a random variable *X* be given a fuzzy linguistic estimate "value is near c". The membership function $\mu_{\tilde{C}}(x)$ of the fuzzy set $\tilde{C}$ ≜"the number is near c" and the corresponding fuzzy number $M = (c, \alpha, \beta)$ are constructed, which sets the boundaries α and β of the interval of possible values of the *X*.
- *Case 2*. Let a random variable *X* be given a fuzzy linguistic estimate "the value is approximately in the range from c to *d*". In this case, a combined Gaussian function and a fuzzy trapezoidal number $M = (c, d, \alpha_1, \beta_2)$. are constructed, where the interval $[\alpha_1, \beta_2]$ specifies the interval of possible values of *X*. Then the value of the middle of the tolerance interval $b = \frac{c+d}{2}$ is calculated and the membership function $\mu_{\tilde{B}}(x)$ is constructed .



The drawing in accordance with distribution law

- The standard beta distribution over the interval $x\epsilon[0,1]$ is given by the density function:

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$$

where

$$B(\alpha,\beta) = \int_0^1 y^{\alpha-1} * (1-y)^{\beta-1} dy$$

Euler's beta function. In this case, the distribution function is expressed through the incomplete beta function:

$$\mathrm{F}(x) = \frac{1}{B(\alpha,\beta)} \int_0^x y^{\alpha-1} * (1-y)^{\beta-1} dy$$

moreover this function is tabulated.

Taking into account that the function $F(x)$ is tabulated, it is proposed to use the Neumann elimination method for drawing random variables on the interval $[a,b]$. The main advantage of this method is its versatility, i.e. applicability for generating random variables having any computable or tabular probability density.

Conclusions

- An approach to modeling random variables on fuzzy intervals of their values is proposed. This approach includes two stages.
- On the first epat, on the basis of a fuzzy linguistic assessment of a random parameter, a fuzzy number is built, which sets a fuzzy interval of its possible values. Depending on the type of assessment, a triangular or trapezoidal fuzzy number is constructed, the fuzzy coefficients of which determine the boundaries of the range of values of the fuzzy value. Such numbers are constructed using the Gaussian membership function.
- At the second stage, a random variable is simulated. The drawing is performed by the Monte Carlo method using Gaussian membership functions and beta distribution. In this case, the drawing of a random variable by the beta distribution function is carried out by the Neumann method.

Thank you for the attention!

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