Dimensionality Reduction of the Criterion Space in Some Optimization Problems

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General statement of the multi-objective optimization problem

$x = \left(x_j, j = \overline{1,n}\right)$

$F = \left(f_1(x), f_2(x), \dots, f_m(x)\right),$ (1)

$g_i(x) \le b_i, i = \overline{1,k},$ (2)

$X \subset R^n$

The point $x^* \in X$ is Pareto optimal if it is valid and there is no other point $x' \neq x^*$, for which

$f_i(x^*) \ge f_i(x'), \forall i = \overline{1,m}.$ (3)

$y^* = \left(f_1(x^*), f_2(x^*), \dots, f_m(x^*)\right).$

$f_i(x) = \sum_{j=1}^{n} c_{ij} x_j \to extr, i = \overline{1,m}, m \ge 2,$ (4)

$\sum_{j=1}^{n} g_{kj} x_j \le b_k, k = \overline{1,p},$ (5)

$x_j \ge 0, j = \overline{1,n}.$ (6)

Clustering of criterion space

Definitions. We call two criteria strongly dependent on some set of admissible alternatives X, if the improvement of the score on one criterion on the set X leads to its improvement by another criterion.

$\pm grad f_i(x)$, $\overline{OC_i}$, $C_i(c_{i1}, c_{i2}, \dots, c_{in}), i = \overline{1,m}.$

$\mu_R\left(\overline{c_i}, \overline{c_j}\right)$ $\mu_R\colon C^2 \to [0,1]$. μ_R^*

$$\mu_R^1\left(\overline{c_i}, \overline{c_j}\right) = \frac{1}{2}\left(1 + \frac{\overline{c_i} \cdot \overline{c_j}}{|\overline{c_i}|\left|\overline{c_j}\right|}\right), \quad (7)$$

$$\mu_R^2\left(\overline{c_i}, \overline{c_j}\right) = \frac{1}{\pi} arccos\left(\frac{\overline{c_i} \cdot \overline{c_j}}{|\overline{c_i}|\left|\overline{c_j}\right|}\right) + 1. \quad (8)$$

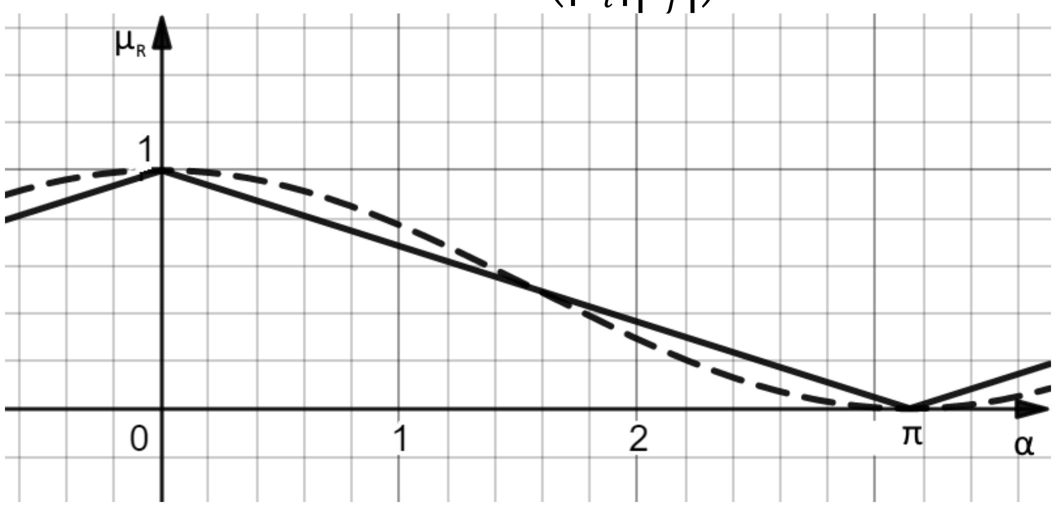


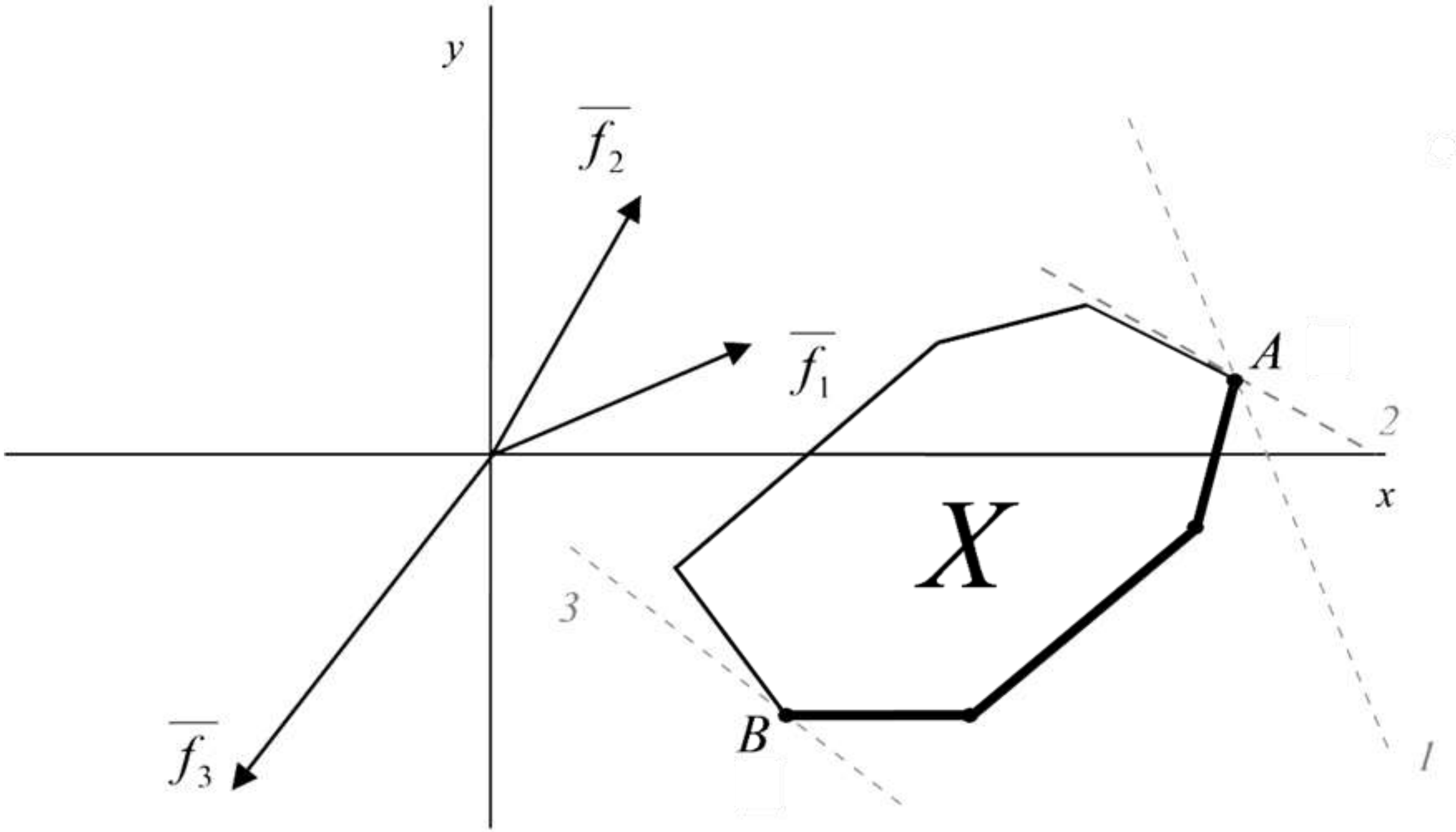
Figure 1: Graphs of similarity measures: μ_R^1 – dotted line, μ_R^2 - solid line

Reducing space of criteria in the weighted sum scalarization method

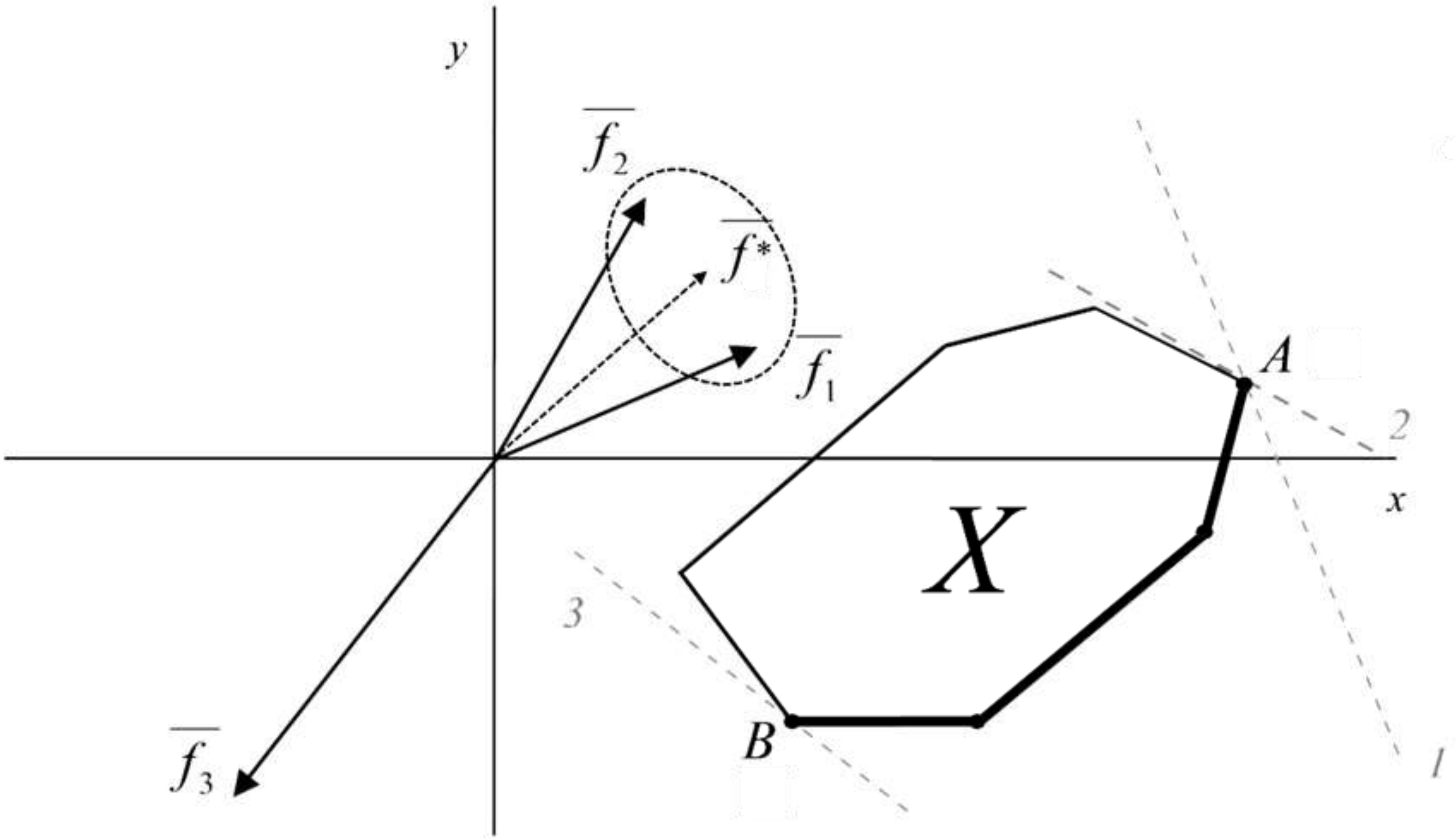
$$F^* = \sum_{i=1}^{m} \alpha_i f_i(x) \to max \quad (11)$$

$$g_k(x) \leq b_k, k = \overline{1,p}, x_j \geq 0, j = \overline{1,n} \quad (12)$$

$$\sum_{i=1}^{m} \alpha_i = 1, \alpha_i \geq 0. \quad (13)$$



- **Figure 2**: Geometric interpretation of the multipurpose linear optimization problem



- **Figure 3**: Geometric interpretation of the result of clustering of criterion space by angular similarity measure

Methods for determining the weights of centroids

$$\beta_i = \frac{|K_i|}{m}, \tag{14}$$

$$\beta_i = \frac{\sum_{j=1}^{z} b_{ij}}{\sum_{i,j}^{z} b_{ij}}, \tag{15}$$

$$F = \sum_{i=1}^{z} \beta_i \cdot \overline{f_i^*}. \tag{16}$$

General scheme of criterion space reduction for the WSS method

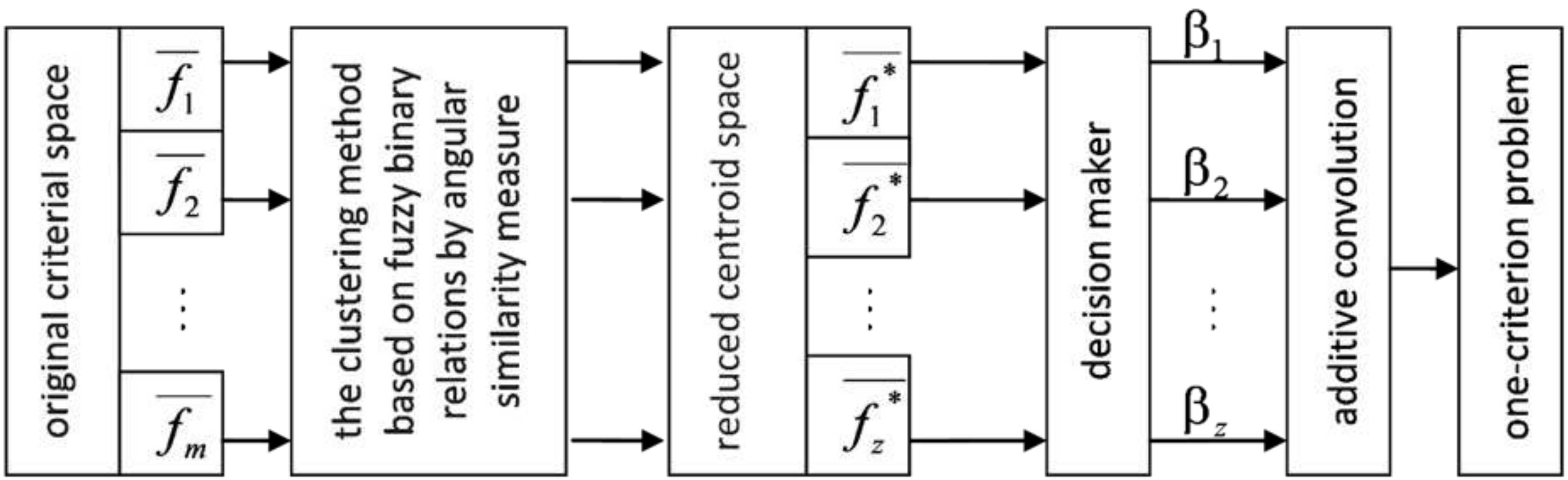


Figure 4: Scheme of reduction of criterion space of MOP at use WSS method

Thanks for your attention