Assortativity of an Elastic Network with Implicit Use of Information about Nodes Degree

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Problem Description

Complex network models:

- Barabasi-Albert model: $\pi_i = \frac{k_i}{\sum_i k_i}$,
- General form of attachment rule:

$$\pi_i = f(i, n, Prop(i), NetProp),$$

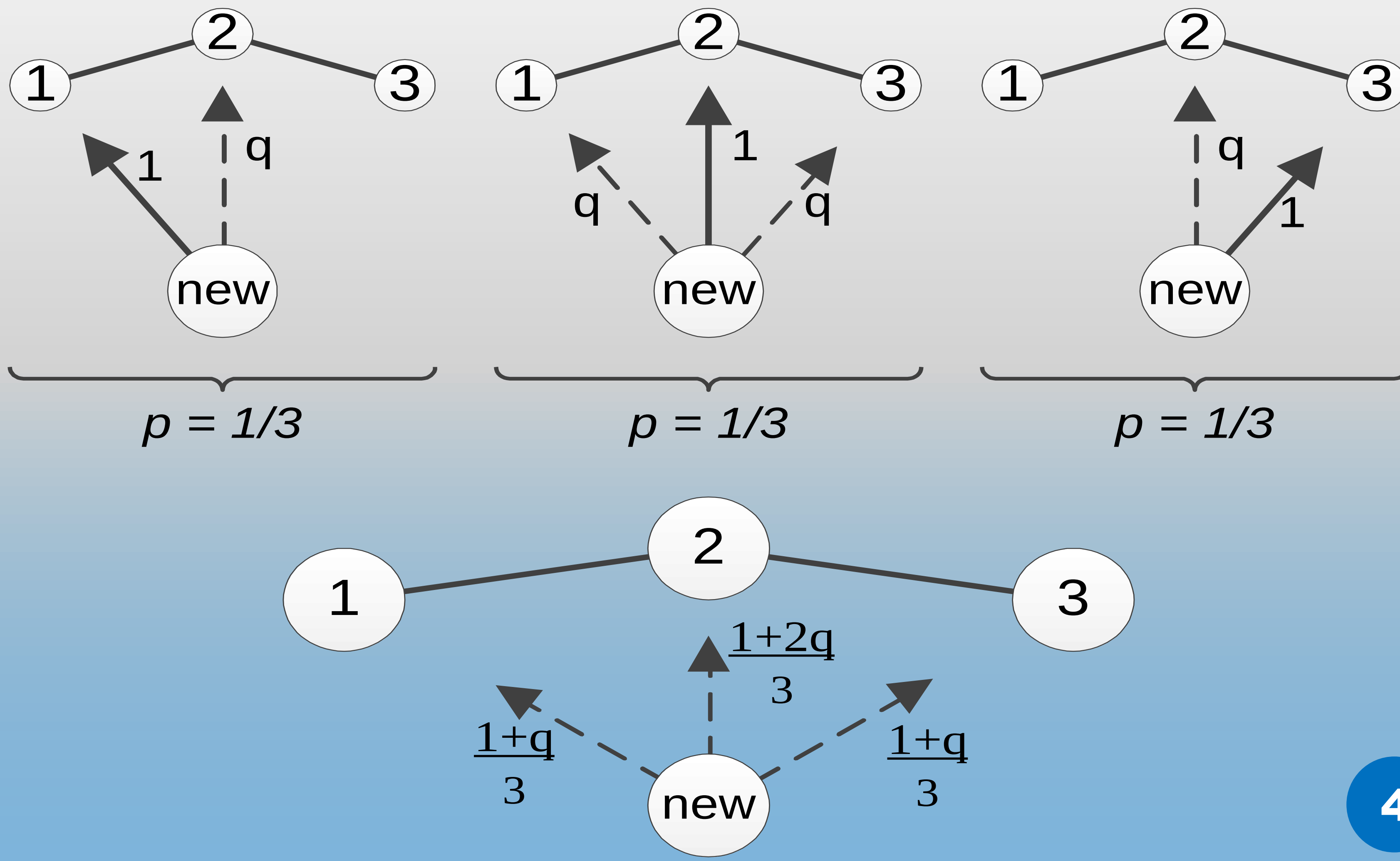
General shortages of traditional models:

- An average degree of nodes is constant in BA- and similar models. Thus, generated networks are very sparse;
- Generated networks are neutral in assortativity;
- <u>Information about nodes degree is used in a direct way</u>, that is far from being natural.

Elastic Mediaton-Driven Attachment Rule

- Each incoming node chooses some existing one uniformly at random. This node is called mediator.
- Then incoming node links to each of the neighbors of this mediator with probability q. This parameter (also called copy-factor) is scale-free and does not depend on any properties of the whole network or its individual nodes.
- The probability π_i for incoming link to attach to node i having degree k_i is $\pi_i = \frac{1+q\cdot k_i}{n}$

Probabilities of Connecting a New Node According to the EMDA Rule



Elasticity of the EMDA-Model

- The network is called elastic if the relative growth rates of the number of links is greater than of nodes. Their ratio is called the elasticity index:

$$\lambda = \frac{\delta L(t)}{\delta n(t)} = \frac{\Delta L(t)}{L(t)} \Big/ \frac{\Delta n(t)}{n(t)} = n \frac{L(n+1) - L(n)}{L(n)}$$

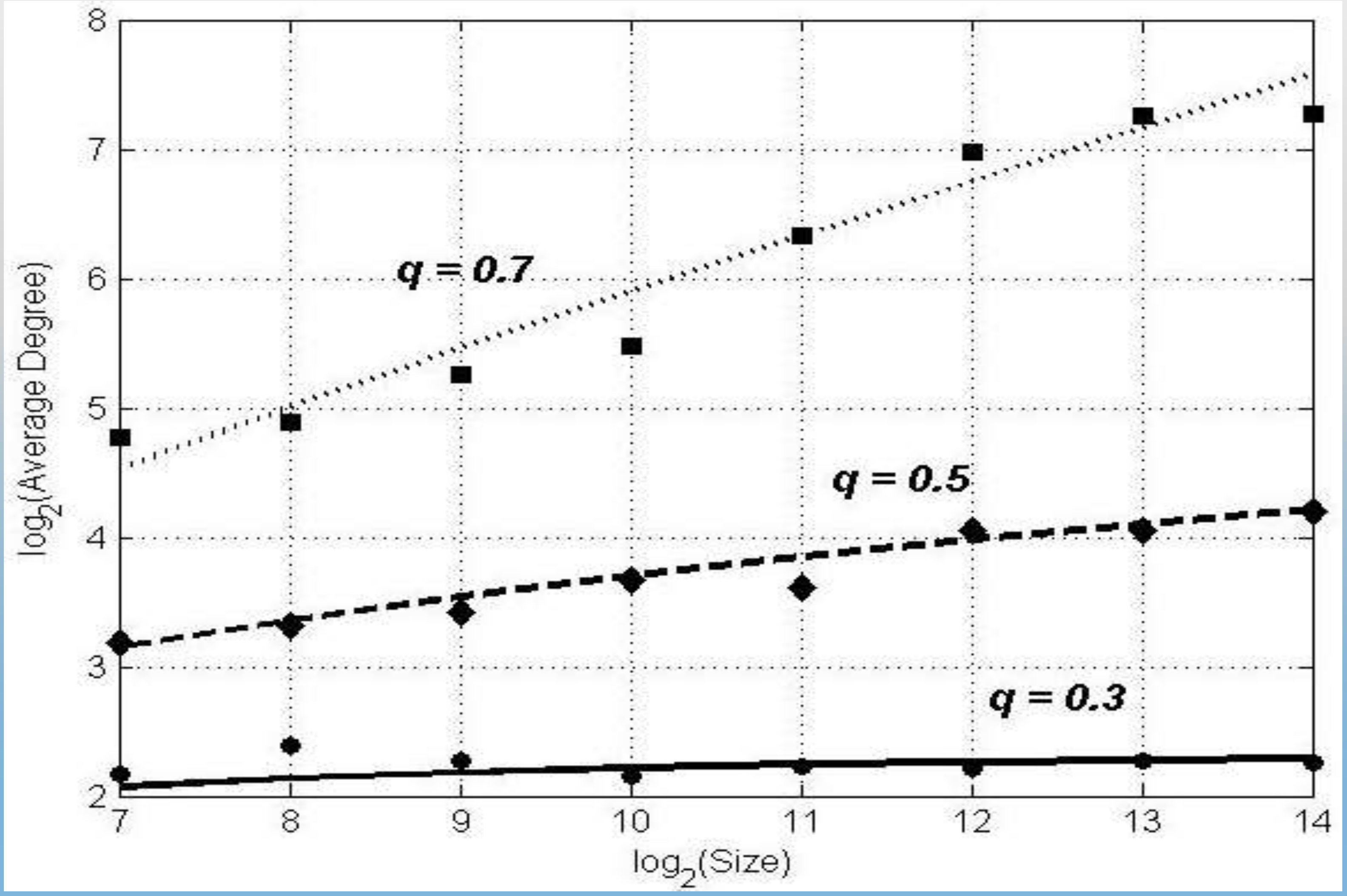
- If so, then the total number of links $L(n)$ in the network and, accordingly, the average degree of nodes $\bar{k}(n)$ grow asymptotically according to the power law.
- Each new node brings more links than their current average: $\Delta L(n) = L(n+1) - L(n) = \lambda \cdot \bar{k}(n)$
- It was proved, that provided EMDA-model generates elastic nets with $\lambda = 2q$.

Grows of the EMDA-Networks

- While $0 < q < 0.5$, the average nodes degree increases to a finite limit $\bar{k}_{lim}$.
- If $0.5 < q < 1$, the average nodes degree grows unboundedly according to an asymptotically power law.
- And for $q = 0.5$ the growth rate is logarithmic:

$$\bar{k}(n) \underset{n\to\infty}{\propto} \begin{cases} 2/(1-2q), & 0<q<0.5 \\ 2\ln(n), & q=0.5 \\ n^{2q-1}, & 0.5<q<1 \end{cases}$$

The Dependence of the Average Degree of Nodes on the Network Size and the Copy Factor



Assortativity of Complex Networks

- An *assortativity coefficient* is defined as correlation coefficient of the nodes by their degrees, or by the adjacency matrix:

$$r = \frac{S_1 N_3 - S_2^2}{S_1 S_3 - S_2^2}.$$

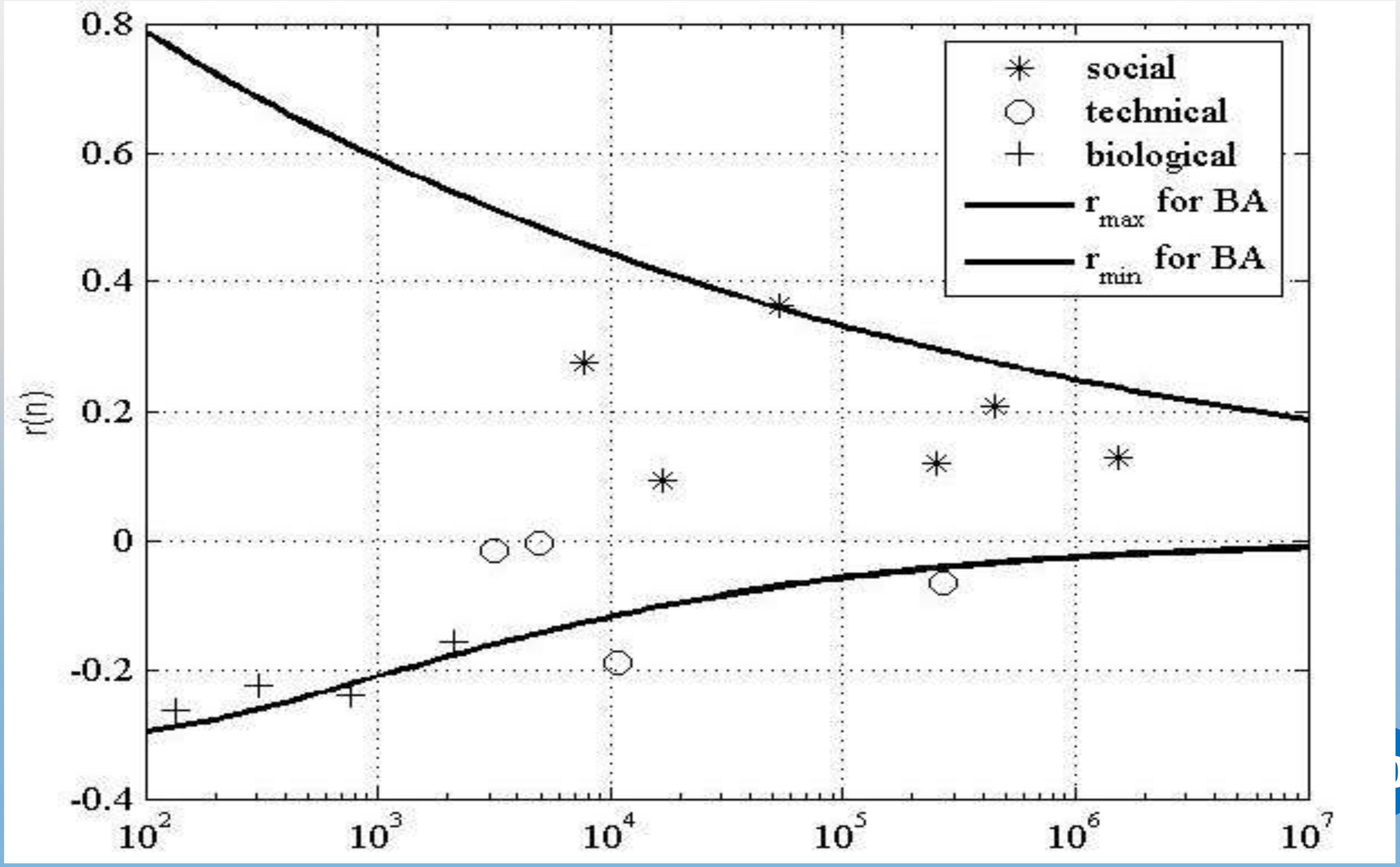
- where

$$S_k = \sum_{i=1}^{n} d_i^k, \quad N_3 = d^T A d = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} d_i d_j$$

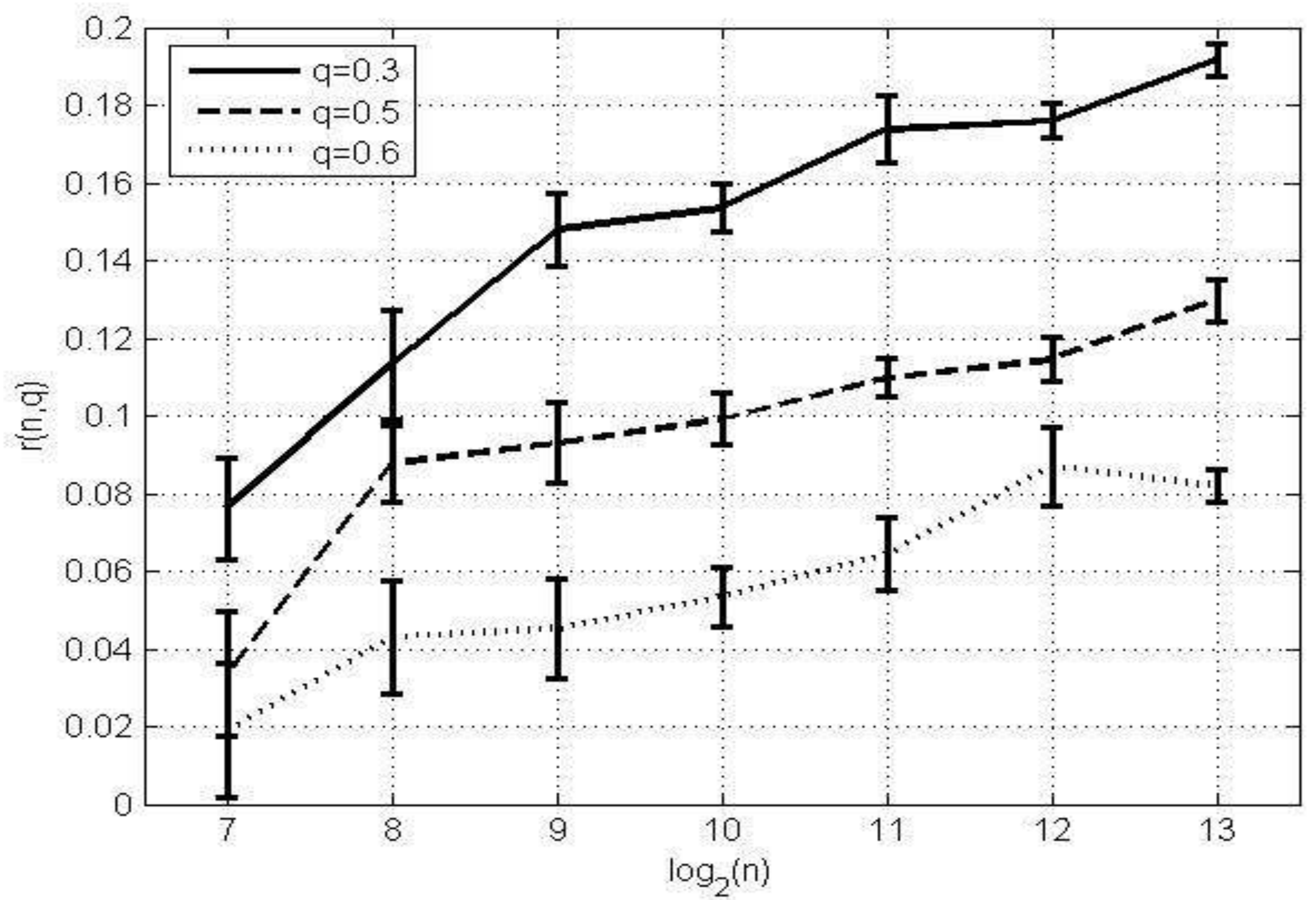
Assortativity of Some Real-World Networks (from [15])

Kind of networks	Network	Size	Assorta-tivity
social	physics coauthorship	52 909	0.363
	biology coauthorship	1 520 251	0.127
	mathematics coauthorship	253 339	0.120
	film actor collaborations	449 913	0.208
	company directors	7 673	0.276
	email address books	16 881	0.092
technological	power grid	4 941	-0.003
	Internet	10 697	-0.189
	World-Wide Web	269 504	-0.067
	software dependencies	3 162	-0.016
biological	protein interactions	2 115	-0.156
	metabolic network	765	-0.240
	neural network	307	-0.226
	marine food web	134	-0.263

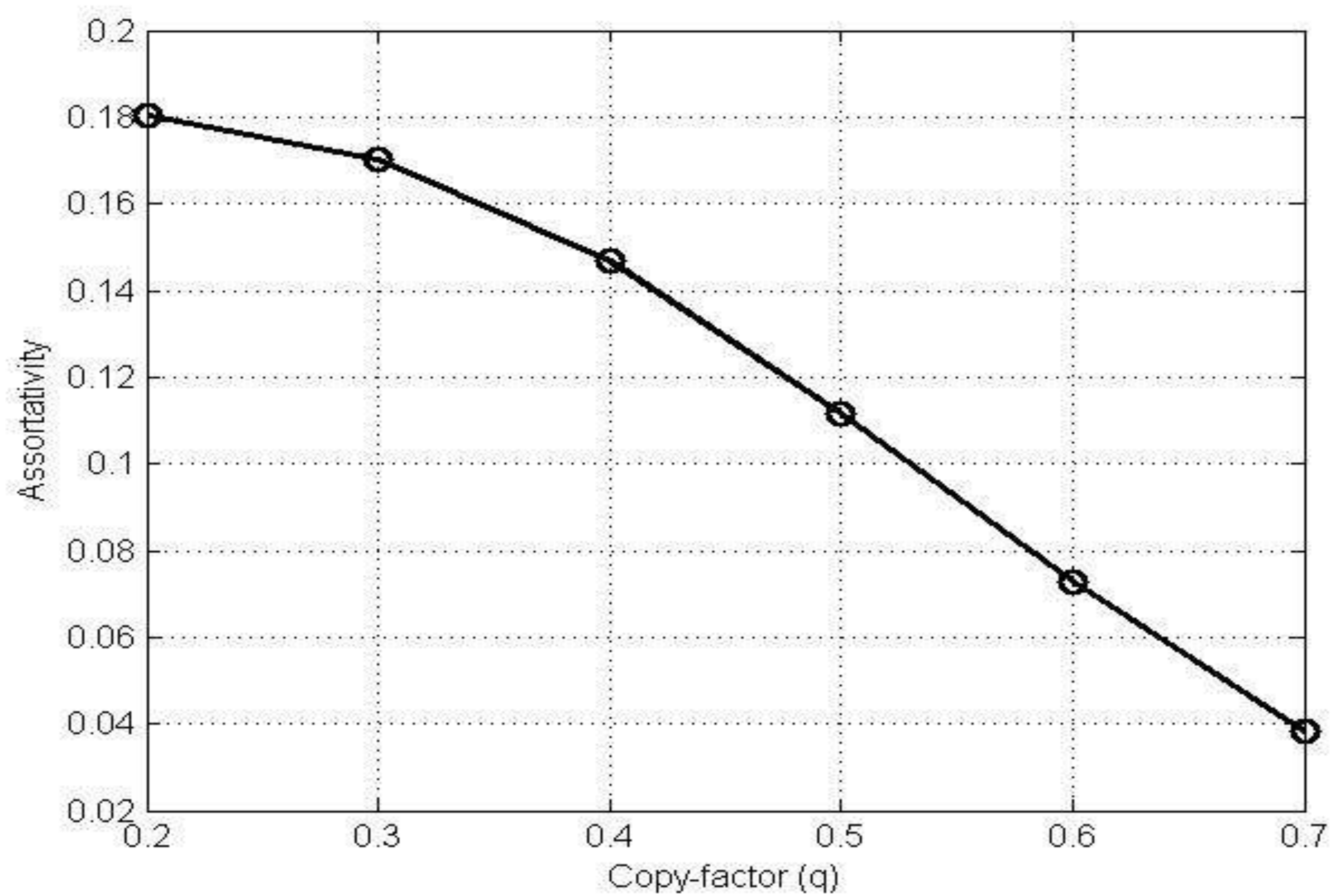
Estimates of the Boundaries of the Assortativity Coefficient for BA-Network



The Dependence of the Assortativity of the Network on its Size and Copy-Factor



The Dependence of the Assortativity of the Network with Fixed Size on the Copy-Factor



Conclusion

- The analysis of dynamic models of complex networks and the corresponding attachment rules is provided. It is shown that the EMDA model is is elastic and does not use node statistics.
- The analysis of the assortative property of networks is carried out. It was found that real-world networks have an explicit property of assortativity (positive or negative), while existing dynamic models generate networks with zero assortativity, i.e. neutral.
- As a result of the provided numerical simulation, it was found that the EMDA algorithm generates networks with significantly positive assor-tativity. Assortativity coefficient decreases with increasing copying factor.
- Thus, it can be argued that the EMDA model reflects the most important properties of social networks: asymptotic scale invariance, implicit use of information about nodes, high density of links, and positive assortativity.
- The theoretical study of the assortativity of the EMDA model is considered as a direction for further research.

Thank you for attention !

Спасибо за внимание !

Děkuji za pozornost !

Dziękuję za uwagę !