Comparative analysis of algorithms for finding the maximum independent set of graphs on quantum and traditional computer

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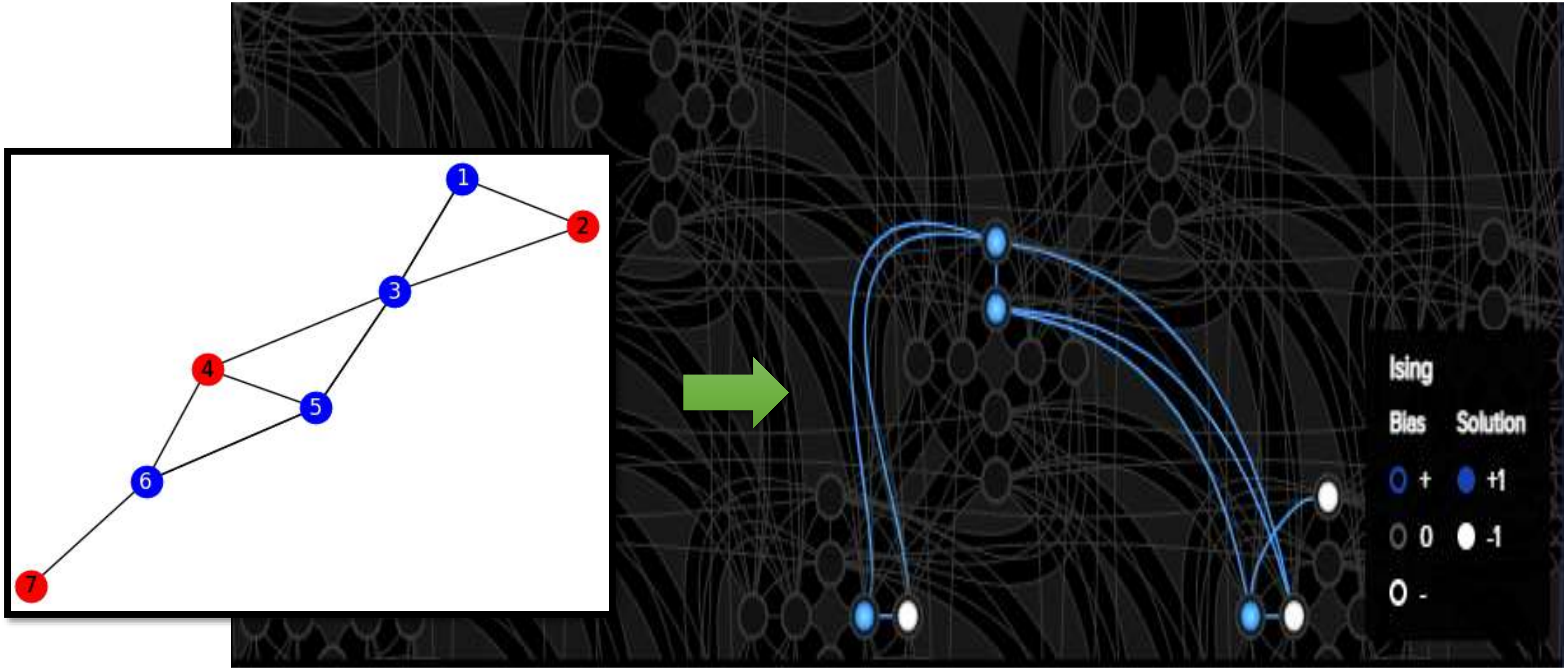
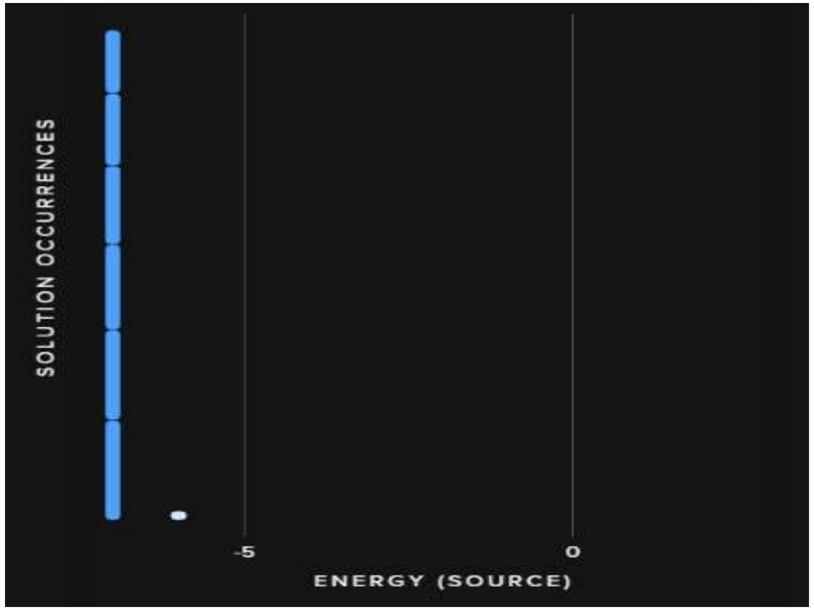
Example of mapping a graph to the qubits of a D-Wave quantum computer to solve the problem of finding the maximum independent set of vertices



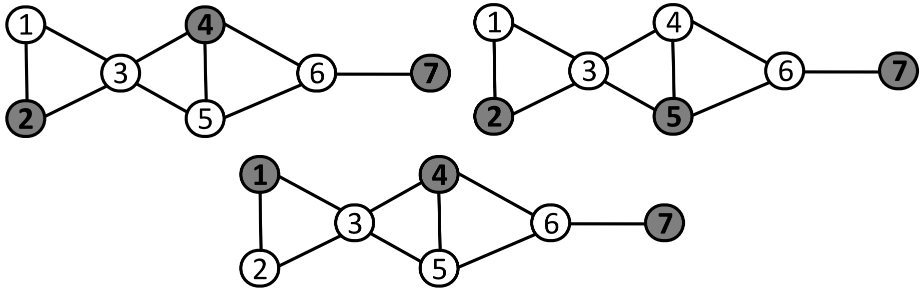
Solution of the problem of finding the Maximum Independent Set of a graph of 7 vertices on a quantum computer by D-wave

The histogram of the frequency of occurrence of solutions (Solution Occurrences) is ordered by the value of energy (Energy / Source). Lower energy values correspond to better solutions.



Probability histogram of the solution of the problem of finding the largest independent set of the graph for 10000 runs, below the energy value "-7". You can also see the erroneous unlikely solution of the problem with the energy value "-6“.

It is clear that for a graph there are several options for solving the problem of finding the maximum independent set.



For example, 6 solutions were found for this graph (Solution Occurrences) for energy value “-7”.

Solving the problem of the largest independent set. Formulation of the problem.

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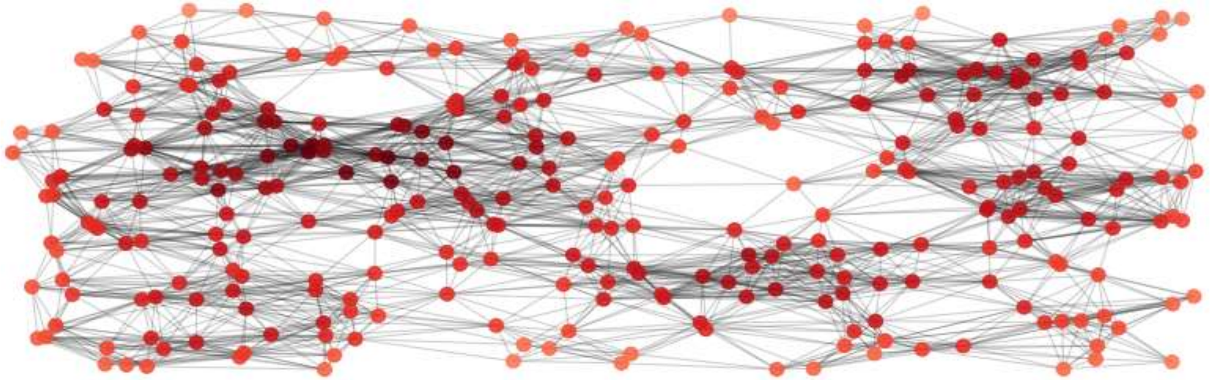
Given a simple undirected graph $G(V, E)$, where V is the set of vertices, E is the set of edges, n is the number of vertices, m is the number of edges, $V = \{1,2, ..., n\}$, $E = \{(u_1, v_1), (u_2, v_2), ..., (u_m, v_m)\}$; $u_i,v_i \in V$; $i = 1, ..., m$. Each vertex i is associated with a variable x_i, which can take the value 0 or 1: $x = (x_1, ..., x_n)$, $x_i \in \{0,1\}$. The function $f(x) = \sum_{i=1}^{n} x_i$ and the constraint on the value of the vector $x = (x_1, ..., x_n)$: $(i, j) \in E \Rightarrow x_i + x_j \leq 1$ are also given.

Thus, the problem of the maximum independent set of a graph (MISG) is formulated as follows:

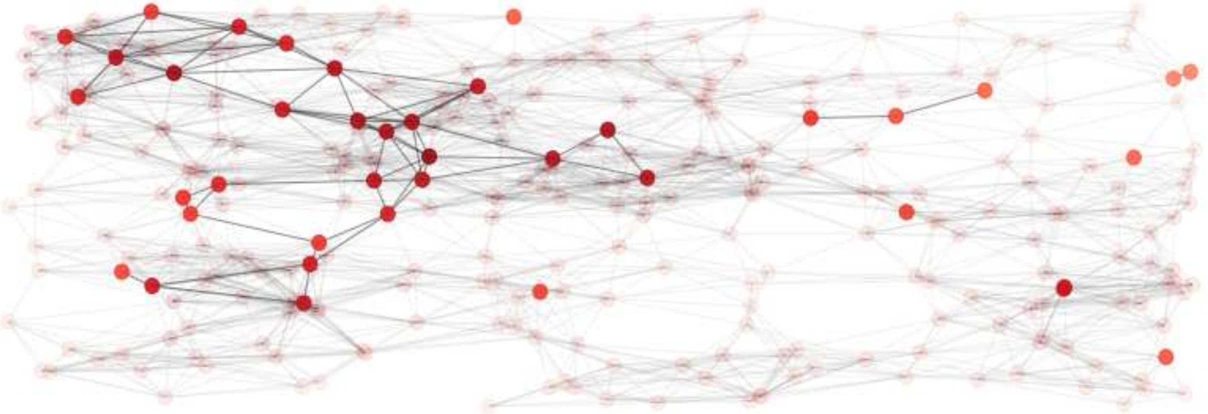
find $max\, f(x) = \sum_{i=1}^{n} x_i$

for constraints $(i, j) \in E \Rightarrow x_i + x_j \leq 1$ $x_i \in \{0,1\}$, $i = 1, ..., n$.

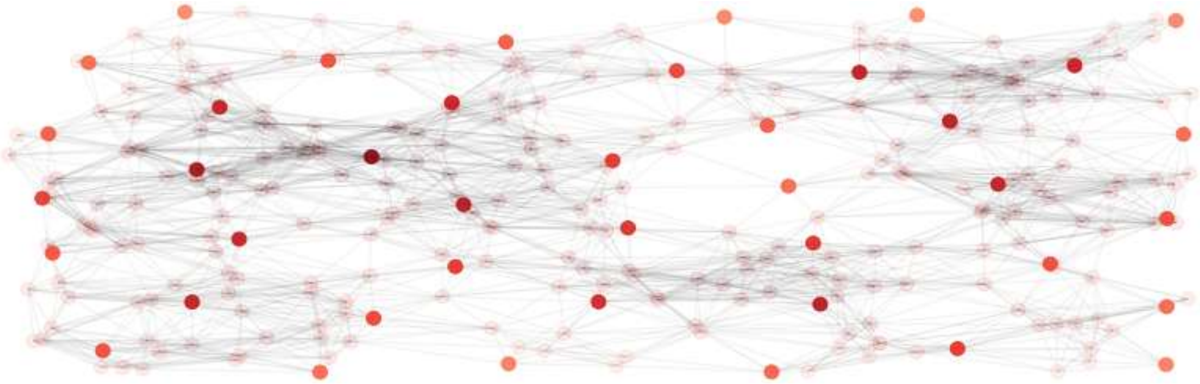
Solving the problem of the Maximum Independent Set in a hybrid quantum-classical cloud



An example of a test random graph with 300 vertices



Example of decomposition on subtasks for graphs with 40 vertices for the problem of finding the Maximum Independent Set of vertices of a graph with 300 vertices (another source graph) by the heuristic algorithm of the firm D-wave



Solution the problem of finding the Maximum Independent Set of graphs for a test graph of 300 vertices

Complete search algorithm. The idea of algorithm:

-search all possible values of the vector x;

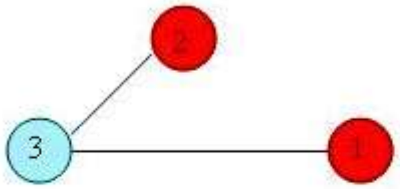
-reject those values of x that do not satisfy the constraint;

-for all remaining x, calculate f (x);

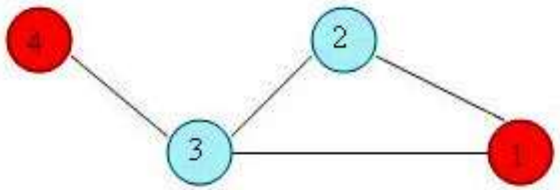
-select the vector x for which the value of f (x) is maximum.

Example of problem solutions
Vertices of the maximum independent set marked in red.

n=3, m=2



n=4, m=3



The current state of quantum computers (QC)

- *Universal QCs* - any quantum arithmetic and logic operations. 10-100 qubits
- *Specialized QCs* - dedicated optimization tasks.

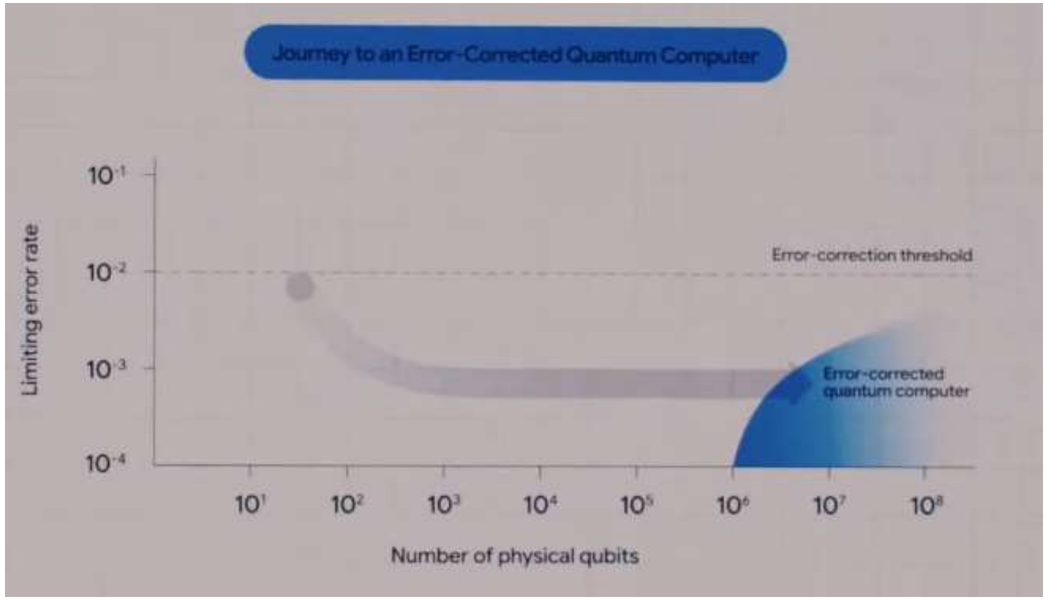
The largest total qubits in QC is about 5,500 qubits.

The fastest QC - Jiuzhang (China, 72 qubits)

The largest number of working qubits - 5436 in the QC of D-wave Advantage ™ (Canada)

Immediate prospects for the development of QC (technological)

- Increasing the number of qubits by more than 1 million and combining several physical qubits into a logical qubit to correct calculation errors



Google's new MEGA supercomputers revealed! (with quantum updates)
https://www.youtube.com/watch?v=xjI80zS3CgQ

Free and paid access to the QC

- IBM provides free and unlimited access to the QC with 10 qubit.
- D-wave provides free limited access to the QC with 5436 qubits.
- There are also many simulators and software libraries: Google Cirq, Microsoft Q #, …
- Paid access to QC is possible: D-wave, IonQ, Rigetti, etc. via Amazon, cost: $ 0.3 task download, cost of one calculation: $ 0.0019 (dwave), $ 0.01 (IonQ), $ 0.00035 (rigetti). It takes at least 100 runs (better results require 1000-10 000 runs) to find the most probable solution. The cost of solving the problem is about $ 4 USA.
- https://aws.amazon.com/ru/braket/pricing/

Conclusions

- Quantum computers have moved from the state of laboratory samples to computing services available over the Internet.
- Today, there is no single quantum computer architecture, and quantum computers are built on different physical principles, which affects the efficiency of high-level algorithms.
- Modern hybrid computing devices that perform quantum annealing in hardware and software are designed to solve complex optimization problems, but they have hardware limitations on the number of qubits and the complexity of their connection.

Conclusions

- Modern quantum-classical (hybrid) computing servers allow you to solve combinatorial optimization problems of medium scale with the speed and dimension required by commercial structures.
- The process of solving the problem of finding the maximum independent set for quantum computers from IBM and D-wave and a quantum-classical cloud server is illustrated.
- The time to find the solution of the problem of finding the maximum independent set for a random graph of 450 vertices and about 17 thousand edges on a personal computer was about 30 seconds, and on a hybrid computer with a quantum coprocessor - 3 seconds.